

Limits by Equation!

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Consider the following limit; we shall solve it without resorting to L'Hopital's rule or power series.

$$\lim_{\theta \rightarrow 0} \frac{\theta - \sin \theta}{\theta^3}$$

To solve this limit, we use equations:

$$\begin{aligned} X &= \lim_{\theta \rightarrow 0} \frac{\theta - \sin \theta}{\theta^3} \\ &= \lim_{\theta \rightarrow 0} \frac{2(\theta/2) - 2 \sin(\theta/2) \cos(\theta/2)}{\theta^3} \\ &= \lim_{\theta \rightarrow 0} \frac{2(\theta/2) - 2 \sin(\theta/2)(1 - 2 \sin^2(\theta/4))}{\theta^3} \\ &= \lim_{\theta \rightarrow 0} \frac{2(\theta/2) - 2 \sin(\theta/2)}{\theta^3} + \lim_{\theta \rightarrow 0} \frac{4 \sin(\theta/2) \sin^2(\theta/4)}{\theta^3} \\ &= X/4 + 1/8 \\ \frac{3}{4}X &= \frac{1}{8} \\ X &= 1/6. \end{aligned}$$

And we are done.