Putnam E.5

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1 Problems

Prof. Loh is in New York this evening because of an event related to the UN General Assembly. Anyone who wants to work on these problems can use Wean 5409, and self-organize a discussion at 5:45pm. Prof. Loh will return to lead the discussion next week (of next week's Tuesday problems) on Tue Sep 29.

Putnam 2003/A1. Let n be a fixed positive integer. How many ways are there to write n as a sum of positive integers, $n = a_1 + a_2 + \cdots + a_k$, with k an arbitrary positive integer and $a_1 \leq a_2 \leq \cdots \leq a_k \leq a_1 + 1$? For example, with $n = 4$ there are four ways: 4, 2+2, 1+1+2, 1+1+1+1.

Putnam 2003/A2. Let $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ be nonnegative real numbers. Show that

$$\begin{aligned}&(a_1 a_2 \cdots a_n)^{1/n} + (b_1 b_2 \cdots b_n)^{1/n} \\ &\leq [(a_1 + b_1)(a_2 + b_2) \cdots (a_n + b_n)]^{1/n}.\end{aligned}$$

Putnam 2003/A3. Find the minimum value of

$$|\sin x + \cos x + \tan x + \cot x + \sec x + \csc x|$$

for real numbers x.