## 21-228 Discrete Mathematics

## Assignment 7

## Due Fri Apr 25

**Notes:** Collaboration is permitted except in the writing stage. Also, please justify every numerical answer with an explanation.

1. Is the following statement true for every positive integer n?

Every graph consisting of two edge-disjoint Hamiltonian paths contains a Hamiltonian cycle.

- 2. Let R(t, t, t) be the smallest integer n such that every 3-coloring of the edges of  $K_n$  contains a monochromatic  $K_t$ . Prove that
  - (a)  $R(t, t, t) \le 27^t$ , and
  - (b) derive an exponential lower bound for the 3-color Ramsey number R(t, t, t). That is, for each t, come up with an  $n_t$ , and a 3-coloring of the edges of  $K_{n_t}$  such that there is no monochromatic  $K_t$ , where  $n_t$  grows exponentially with t.

**Note:** I can do this with  $n_t = 3^{t/2}$  for every even t (which is sufficient), but it is OK if you prove this with a different  $n_t$ , as long as yours grows exponentially with t.

3. There are 100 cities in a country, called "1", "2", ..., "100". The government would like to connect them all with a network, and it costs  $\max\{|i-j|,4\}$  to lay a wire between cities "i" and "j". (There is a minimum cost of 4 per wire.) Another company comes along, offering to create microwave links between city pairs at a cost of 2 each, with the catch that their technology can only connect city pairs of the form ("i", "2i"). For example, it would cost 30 to lay a wire between cities 31 and 61, 4 to lay a wire between cities 31 and 32, and 2 to microwave-link 30 and 60. What is the minimum cost to create this connected network?