Putnam E.14

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3 December 2024

1 Problems

Putnam 2020/A1. How many positive integers N satisfy all of the following three conditions?

- (i) N is divisible by 2020.
- (ii) N has at most 2020 decimal digits.
- (iii) The decimal digits of N are a string of consecutive ones followed by a string of consecutive zeros.

Putnam 2020/A2. Let k be a nonnegative integer. Evaluate

$$\sum_{j=0}^{k} 2^{k-j} \binom{k+j}{j}.$$

Putnam 2020/A3. Let $a_0 = \pi/2$, and let $a_n = \sin(a_{n-1})$ for $n \ge 1$. Determine whether

$$\sum_{n=1}^{\infty} a_n^2$$

converges.