

# Putnam $\Sigma.10$

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## 1 Problems

**Putnam 2000/A4.** Show that the improper integral

$$\lim_{B \rightarrow \infty} \int_0^B \sin(x) \sin(x^2) dx$$

converges.

**Putnam 2000/A5.** Three distinct points with integer coordinates lie in the plane on a circle of radius  $r > 0$ . Show that two of these points are separated by a distance of at least  $r^{1/3}$ .

**Putnam 2000/A6.** Let  $f(x)$  be a polynomial with integer coefficients. Define a sequence  $a_0, a_1, \dots$  of integers such that  $a_0 = 0$  and  $a_{n+1} = f(a_n)$  for all  $n \geq 0$ . Prove that if there exists a positive integer  $m$  for which  $a_m = 0$  then either  $a_1 = 0$  or  $a_2 = 0$ .