

Putnam $\Sigma.9$

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SATURDAY Oct 28

1 Problems

This weekend's meeting will be on **SATURDAY October 28** instead of the usual Sunday. We'll meet in Wean 5302, which we have borrowed from CMIMC. Technically, CMIMC's reservation only starts at 4pm, so if anyone else is there already, you can safely ask them to please move out at 4pm. Then the room is ours!

Putnam 2001/B4. Let S denote the set of rational numbers different from $\{-1, 0, 1\}$. Define $f : S \rightarrow S$ by $f(x) = x - 1/x$. Prove or disprove that

$$\bigcap_{n=1}^{\infty} f^{(n)}(S) = \emptyset,$$

where $f^{(n)}$ denotes f composed with itself n times.

Putnam 2001/B5. Let a and b be real numbers in the interval $(0, 1/2)$, and let g be a continuous real-valued function such that $g(g(x)) = ag(x) + bx$ for all real x . Prove that $g(x) = cx$ for some constant c .

Putnam 2001/B6. Assume that $(a_n)_{n \geq 1}$ is an increasing sequence of positive real numbers such that $\lim a_n/n = 0$. Must there exist infinitely many positive integers n such that $a_{n-i} + a_{n+i} < 2a_n$ for $i = 1, 2, \dots, n-1$?