

# Putnam E.2

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## 1 Problems

**2012/A1.** Let  $d_1, d_2, \dots, d_{12}$  be real numbers in the open interval  $(1, 12)$ . Show that there exist distinct indices  $i, j, k$  such that  $d_i, d_j, d_k$  are the side lengths of an acute triangle.

**2012/A2.** Let  $*$  be a commutative and associative binary operation on a set  $S$ . Assume that for every  $x$  and  $y$  in  $S$ , there exists  $z$  in  $S$  such that  $x * z = y$ . (This  $z$  may depend on  $x$  and  $y$ .) Show that if  $a, b, c$  are in  $S$  and  $a * c = b * c$ , then  $a = b$ .

**2012/A3.** Let  $f : [-1, 1] \rightarrow \mathbb{R}$  be a continuous function such that

- (i)  $f(x) = \frac{2-x^2}{2} f\left(\frac{x^2}{2-x^2}\right)$  for every  $x$  in  $[-1, 1]$ ,
- (ii)  $f(0) = 1$ , and
- (iii)  $\lim_{x \rightarrow 1^-} \frac{f(x)}{\sqrt{1-x}}$  exists and is finite.

Prove that  $f$  is unique, and express  $f(x)$  in closed form.