

# Putnam $\Sigma.9$

Po-Shen Loh

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## 1 Problems

**Putnam 2010/A4.** Prove that for each positive integer  $n$ , the number  $10^{10^{10^n}} + 10^{10^n} + 10^n - 1$  is not prime.

**Putnam 2010/A5.** Let  $G$  be a group, with operation  $*$ . Suppose that

- (i)  $G$  is a subset of  $\mathbb{R}^3$  (but  $*$  need not be related to addition of vectors);
- (ii) For each  $\mathbf{a}, \mathbf{b} \in G$ , either  $\mathbf{a} \times \mathbf{b} = \mathbf{a} * \mathbf{b}$  or  $\mathbf{a} \times \mathbf{b} = \mathbf{0}$  (or both), where  $\times$  is the usual cross product in  $\mathbb{R}^3$ .

Prove that  $\mathbf{a} \times \mathbf{b} = \mathbf{0}$  for all  $\mathbf{a}, \mathbf{b} \in G$ .

**Putnam 2010/A6.** Let  $f : [0, \infty) \rightarrow \mathbb{R}$  be a strictly decreasing continuous function such that  $\lim_{x \rightarrow \infty} f(x) = 0$ . Prove that  $\int_0^\infty \frac{f(x) - f(x+1)}{f(x)} dx$  diverges.