Putnam Σ.7 Break Edition

Po-Shen Loh

16 October 2022

1 Problems

Putnam 2008/B4. Let p be a prime number. Let $h(x)$ be a polynomial with integer coefficients such that $h(0), h(1), \ldots, h(p^2-1)$ are distinct modulo p^2. Show that $h(0), h(1), \ldots, h(p^3-1)$ are distinct modulo p^3.

Putnam 2008/B5. Find all continuously differentiable functions $f : \mathbb{R} \to \mathbb{R}$ such that for every rational number q, the number $f(q)$ is rational and has the same denominator as q. (The denominator of a rational number q is the unique positive integer b such that $q = a/b$ for some integer a with $\gcd(a, b) = 1$.) (Note: gcd means greatest common divisor.)

Putnam 2008/B6. Let n and k be positive integers. Say that a permutation σ of $\{1, 2, \ldots, n\}$ is *k-limited* if $|\sigma(i) - i| \leq k$ for all i. Prove that the number of k-limited permutations of $\{1, 2, \ldots, n\}$ is odd if and only if $n \equiv 0$ or $1 \pmod{2k+1}$.