Putnam E.11

Po-Shen Loh

15 November 2022

1   Problems

Putnam 2000/A1. Let A be a positive real number. What are the possible values of $\sum_{j=0}^{\infty} x_j^2$, given that $x_0, x_1, \ldots$ are positive numbers for which $\sum_{j=0}^{\infty} x_j = A$?

Putnam 2000/A2. Prove that there exist infinitely many integers n such that $n, n+1, n+2$ are each the sum of the squares of two integers. [Example: $0 = 0^2 + 0^2$, $1 = 0^2 + 1^2$, $2 = 1^2 + 1^2$.]

Putnam 2000/A3. The octagon $P_1 P_2 P_3 P_4 P_5 P_6 P_7 P_8$ is inscribed in a circle, with the vertices around the circumference in the given order. Given that the polygon $P_1 P_3 P_5 P_7$ is a square of area 5, and the polygon $P_2 P_4 P_6 P_8$ is a rectangle of area 4, find the maximum possible area of the octagon.