

# Putnam E.5

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27 September 2022

## 1 Problems

**Putnam 2003/A1.** Let  $n$  be a fixed positive integer. How many ways are there to write  $n$  as a sum of positive integers,  $n = a_1 + a_2 + \cdots + a_k$ , with  $k$  an arbitrary positive integer and  $a_1 \leq a_2 \leq \cdots \leq a_k \leq a_1 + 1$ ? For example, with  $n = 4$  there are four ways:  $4$ ,  $2+2$ ,  $1+1+2$ ,  $1+1+1+1$ .

**Putnam 2003/A2.** Let  $a_1, a_2, \dots, a_n$  and  $b_1, b_2, \dots, b_n$  be nonnegative real numbers. Show that

$$\begin{aligned} & (a_1 a_2 \cdots a_n)^{1/n} + (b_1 b_2 \cdots b_n)^{1/n} \\ & \leq [(a_1 + b_1)(a_2 + b_2) \cdots (a_n + b_n)]^{1/n}. \end{aligned}$$

**Putnam 2003/A3.** Find the minimum value of

$$|\sin x + \cos x + \tan x + \cot x + \sec x + \csc x|$$

for real numbers  $x$ .