

# Putnam E.2

Po-Shen Loh

6 September 2022

## 1 Problems

**Putnam 2005/A1.** Show that every positive integer is a sum of one or more numbers of the form  $2^r 3^s$ , where  $r$  and  $s$  are nonnegative integers and no summand divides another. (For example,  $23 = 9 + 8 + 6$ .)

**Putnam 2005/A2.** Let  $\mathbf{S} = \{(a, b) \mid a = 1, 2, \dots, n, b = 1, 2, 3\}$ . A *rook tour* of  $\mathbf{S}$  is a polygonal path made up of line segments connecting points  $p_1, p_2, \dots, p_{3n}$  in sequence such that

- (i)  $p_i \in \mathbf{S}$ ,
- (ii)  $p_i$  and  $p_{i+1}$  are a unit distance apart, for  $1 \leq i < 3n$ ,
- (iii) for each  $p \in \mathbf{S}$  there is a unique  $i$  such that  $p_i = p$ .

How many rook tours are there that begin at  $(1, 1)$  and end at  $(n, 1)$ ?

**Putnam 2005/A3.** Let  $p(z)$  be a nonconstant polynomial of degree  $n$ , all of whose zeros have absolute value 1 in the complex plane. Put  $g(z) = p(z)/z^{n/2}$ . Show that all zeros of  $g'(z) = 0$  have absolute value 1.