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title: "Putnam Σ.14"

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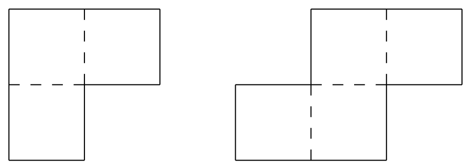
1 Problems

Putnam 2016/A4. Consider a $(2m-1) \times (2n-1)$ rectangular region, where m and n are integers such that $m, n \geq 4$. This region is to be tiled using tiles of the two types shown:



(The dotted lines divide the tiles into 1×1 squares.) The tiles may be rotated and reflected, as long as their sides are parallel to the sides of the rectangular region. They must all fit within the region, and they must cover it completely without overlapping.

What is the minimum number of tiles required to tile the region?

Putnam 2016/A5. Suppose that G is a finite group generated by the two elements g and h, where the order of g is odd. Show that every element of G can be written in the form

$$g^{m_1} h^{n_1} g^{m_2} h^{n_2} \cdots g^{m_r} h^{n_r}$$

with $1 \leq r \leq |G|$ and $m_1, n_1, m_2, n_2, \ldots, m_r, n_r \in \{-1, 1\}$. (Here $|G|$ is the number of elements of G.)

Putnam 2016/A6. Find the smallest constant C such that for every real polynomial $P(x)$ of degree 3 that has a root in the interval $[0, 1]$,

$$\int_0^1 |P(x)| \, dx \leq C \max_{x \in [0,1]} |P(x)| \, .$$