Putnam Σ.12

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14 November 2021

1 Problems

Putnam 2009/A4. Let S be a set of rational numbers such that

(a) $0 \in S$;

(b) If $x \in S$ then $x+1 \in S$ and $x-1 \in S$; and

(c) If $x \in S$ and $x \notin \{0,1\}$, then $\frac{1}{x(x-1)} \in S$.

Must S contain all rational numbers?

Putnam 2009/A5. Is there a finite abelian group G such that the product of the orders of all its elements is 2^{2009}?

Putnam 2009/A6. Let $f : [0,1]^2 \to \mathbb{R}$ be a continuous function on the closed unit square such that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist and are continuous on the interior $(0,1)^2$. Let $a = \int_0^1 f(0,y)\,dy$, $b = \int_0^1 f(1,y)\,dy$, $c = \int_0^1 f(x,0)\,dx$, $d = \int_0^1 f(x,1)\,dx$. Prove or disprove: There must be a point (x_0, y_0) in $(0,1)^2$ such that

$$\frac{\partial f}{\partial x}(x_0,y_0) = b - a \quad \text{and} \quad \frac{\partial f}{\partial y}(x_0,y_0) = d - c.$$