Putnam Σ.10

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1 Problems

It is getting cold outside, so we'll go together to Orient Express and order once we get there.

Putnam 2015/B4. Let T be the set of all triples (a, b, c) of positive integers for which there exist triangles with side lengths a, b, c. Express

$$\sum_{(a,b,c) \in T} \frac{2^a}{3^b 5^c}$$

as a rational number in lowest terms.

Putnam 2015/B5. Let P_n be the number of permutations π of $\{1, 2, \ldots, n\}$ such that

$$|i - j| = 1 \text{ implies } |\pi(i) - \pi(j)| \leq 2$$

for all i, j in $\{1, 2, \ldots, n\}$. Show that for $n \geq 2$, the quantity

$$P_{n+5} - P_{n+4} - P_{n+3} + P_n$$

does not depend on n, and find its value.

Putnam 2015/B6. For each positive integer k, let $A(k)$ be the number of odd divisors of k in the interval $[1, \sqrt{2k})$. Evaluate

$$\sum_{k=1}^{\infty} (-1)^{k-1} \frac{A(k)}{k}.$$