Putnam E.14

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1 Problems

Putnam 2008/B1. What is the maximum number of rational points that can lie on a circle in $\mathbb{R}^2$ whose center is not a rational point? (A *rational point* is a point both of whose coordinates are rational numbers.)

Putnam 2008/B2. Let $F_0(x) = \ln x$. For $n \geq 0$ and $x > 0$, let $F_{n+1}(x) = \int_0^x F_n(t)dt$. Evaluate

$$\lim_{n\to\infty} \frac{n!F_n(1)}{\ln n}.$$

Putnam 2008/B3. What is the largest possible radius of a circle contained in a 4-dimensional hypercube of side length 1?