Putnam E.8

Po-Shen Loh

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1 Problems

Putnam 2007/B1. Let f be a nonconstant polynomial with positive integer coefficients. Prove that if n is a positive integer, then $f(n)$ divides $f(f(n)+1)$ if and only if $n = 1$.

Putnam 2007/B2. Suppose that $f : [0,1] \to \mathbb{R}$ has a continuous derivative and that $\int_0^1 f(x)\,dx = 0$. Prove that for every $\alpha \in (0,1)$,

$$\left| \int_0^\alpha f(x)\,dx \right| \le \frac{1}{8} \max_{0 \le x \le 1} |f'(x)|.$$

Putnam 2007/B3. Let $x_0 = 1$ and for $n \ge 0$, let $x_{n+1} = 3x_n + \lfloor x_n\sqrt{5} \rfloor$. In particular, $x_1 = 5$, $x_2 = 26$, $x_3 = 136$, $x_4 = 712$. Find a closed-form expression for x_{2007}. ($\lfloor a \rfloor$ means the largest integer $\le a$.)