21-228 Discrete Mathematics Exam 3 Alternate Time

April 30, 2021

Problem	Score
1	
2	
3	
4	
5	
Total	

Name: ______________________________

This 50-minute exam is open-notes, in the sense that you may use anything you have written yourself. No calculators are permitted. Please write your answers in the space provided, and indicate clearly on the front of a page if you use the back of that page for additional space. Every numerical answer must be justified with an explanation. You may use any theorem which was stated in class without reproving it. Each problem is worth 10 points.

1. Prove that every tree which contains exactly 3 vertices of degree 5 necessarily contains at least 11 leaves.

2. State an algorithm for finding a **maximum** spanning tree in weighted connected graphs, and prove why it works. You may use any result which was proven in class without reproving it again, by stating it here and then using it in that form. (However, you cannot state a different result from what we proved in class, and claim that a "similar proof" would establish that different result.)

3. How many n-vertex trees with vertices labeled $\{1, 2, \ldots, n\}$ are there where the vertex labeled 2 is a leaf?

4. Suppose that a connected n-vertex graph has no Hamiltonian cycle. Prove that every longest path in the graph has the property that at least one of its endpoints has degree less than $n/2$. (In any given graph, there can be several longest paths, because there could be several different paths with that same longest length.)

5. Prove that no matter how the edges of the complete graph on 17 vertices are colored using red, green, or blue, there is always a monochromatic K_3 (i.e., 3 vertices for which each of the 3 edges among them are the same color).