Putnam Σ.10

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1 Problems

Putnam 1996/A4. Let S be the set of ordered triples (a, b, c) of distinct elements of a finite set A. Suppose that

1. $(a, b, c) \in S$ if and only if $(b, c, a) \in S$;
2. $(a, b, c) \in S$ if and only if $(c, b, a) \notin S$;
3. (a, b, c) and (c, d, a) are both in S if and only if (b, c, d) and (d, a, b) are both in S.

Prove that there exists a one-to-one function g from A to R such that $g(a) < g(b) < g(c)$ implies $(a, b, c) \in S$. Note: R is the set of real numbers.

Putnam 1996/A5. If p is a prime number greater than 3 and $k = \lfloor 2p/3 \rfloor$, prove that the sum

$$\binom{p}{1} + \binom{p}{2} + \cdots + \binom{p}{k}$$

of binomial coefficients is divisible by p^2.

Putnam 1996/A6. Let $c > 0$ be a constant. Give a complete description, with proof, of the set of all continuous functions $f : R \to R$ such that $f(x) = f(x^2 + c)$ for all $x \in R$. Note that R denotes the set of real numbers.