# Putnam Σ.6

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## 1 Problems

**Putnam 1997/A4.** Let $G$ be a group with identity $e$ and $\phi : G \to G$ a function such that

$$\phi(g_1)\phi(g_2)\phi(g_3) = \phi(h_1)\phi(h_2)\phi(h_3)$$

whenever $g_1 g_2 g_3 = e = h_1 h_2 h_3$. Prove that there exists an element $a \in G$ such that $\psi(x) = a\phi(x)$ is a homomorphism (i.e. $\psi(xy) = \psi(x)\psi(y)$ for all $x, y \in G$).

**Putnam 1997/A5.** Let $N_n$ denote the number of ordered $n$-tuples of positive integers $(a_1, a_2, \ldots, a_n)$ such that $1/a_1 + 1/a_2 + \ldots + 1/a_n = 1$. Determine whether $N_{10}$ is even or odd.

**Putnam 1997/A6.** For a positive integer $n$ and any real number $c$, define $x_k$ recursively by $x_0 = 0$, $x_1 = 1$, and for $k \geq 0$,

$$x_{k+2} = \frac{cx_{k+1} - (n-k)x_k}{k+1}.$$

Fix $n$ and then take $c$ to be the largest value for which $x_{n+1} = 0$. Find $x_k$ in terms of $n$ and $k$, $1 \leq k \leq n$.