Putnam E.14

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1 Problems

Putnam 1981/A1. Let the largest power of 5 dividing $1^1 2^2 3^3 \cdots n^n$ be $5^{f(n)}$. What is

$$\lim_{n\to\infty} \frac{f(n)}{n^2}?$$

Putnam 1981/A2. We can label the squares of an 8×8 chess board from from 1 to 64 in 64! different ways. For each way, we compute D, the largest difference between the labels of two squares which are adjacent (sharing an edge or a corner). What is the smallest possible D?

Putnam 1981/A3. Evaluate:

$$\lim_{k\to\infty} e^{-k} \int_0^k \int_0^k \frac{e^x - e^y}{x - y} dx dy.$$