

# Putnam E.4

Po-Shen Loh

22 Sep 2020

## 1 Problems

**Putnam 1987/B1.** Evaluate

$$\int_2^4 \frac{\sqrt{\ln(9-x)}dx}{\sqrt{\ln(9-x)} + \sqrt{\ln(x+3)}}.$$

**Putnam 1987/B2.** Let  $r$ ,  $s$ , and  $t$  be integers with  $0 \leq r$ ,  $0 \leq s$ , and  $r + s \leq t$ . Prove that

$$\frac{\binom{s}{0}}{\binom{t}{r}} + \frac{\binom{s}{1}}{\binom{t}{r+1}} + \cdots + \frac{\binom{s}{s}}{\binom{t}{r+s}} = \frac{t+1}{(t+1-s)\binom{t-s}{r}}.$$

**Putnam 1987/B3.** Let  $F$  be a field in which  $1 + 1 \neq 0$ . Show that the set of solutions to the equation  $x^2 + y^2 = 1$  with  $x$  and  $y$  in  $F$  is given by  $(x, y) = (1, 0)$  and

$$(x, y) = \left( \frac{r^2 - 1}{r^2 + 1}, \frac{2r}{r^2 + 1} \right),$$

where  $r$  runs through the elements of  $F$  such that  $r^2 \neq -1$ .