Putnam Σ.7

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1 Problems

Putnam 2001/B4. Let S denote the set of rational numbers different from $\{-1, 0, 1\}$. Define $f : S \to S$ by $f(x) = x - 1/x$. Prove or disprove that

$$\bigcap_{n=1}^{\infty} f^{(n)}(S) = \emptyset,$$

where $f^{(n)}$ denotes f composed with itself n times.

Putnam 2001/B5. Let a and b be real numbers in the interval $(0, 1/2)$, and let g be a continuous real-valued function such that $g(g(x)) = ag(x) + bx$ for all real x. Prove that $g(x) = cx$ for some constant c.

Putnam 2001/B6. Assume that $(a_n)_{n \geq 1}$ is an increasing sequence of positive real numbers such that $\lim a_n/n = 0$. Must there exist infinitely many positive integers n such that $a_{n-i} + a_{n+i} < 2a_n$ for $i = 1, 2, \ldots, n-1$?