

# Putnam $\Sigma.5$

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## 1 Problems

**Putnam 2002/B4.** An integer  $n$ , unknown to you, has been randomly chosen in the interval  $[1, 2002]$  with uniform probability. Your objective is to select  $n$  in an **odd** number of guesses. After each incorrect guess, you are informed whether  $n$  is higher or lower, and you **must** guess an integer on your next turn among the numbers that are still feasibly correct. Show that you have a strategy so that the chance of winning is greater than  $2/3$ .

**Putnam 2002/B5.** A palindrome in base  $b$  is a positive integer whose base- $b$  digits read the same backwards and forwards; for example, 2002 is a 4-digit palindrome in base 10. Note that 200 is not a palindrome in base 10, but it is the 3-digit palindrome 242 in base 9, and 404 in base 7. Prove that there is an integer which is a 3-digit palindrome in base  $b$  for at least 2002 different values of  $b$ .

**Putnam 2002/B6.** Let  $p$  be a prime number. Prove that the determinant of the matrix

$$\begin{pmatrix} x & y & z \\ x^p & y^p & z^p \\ x^{p^2} & y^{p^2} & z^{p^2} \end{pmatrix}$$

is congruent modulo  $p$  to a product of polynomials of the form  $ax + by + cz$ , where  $a, b, c$  are integers. (We say two integer polynomials are congruent modulo  $p$  if corresponding coefficients are congruent modulo  $p$ .)