Putnam E.15

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1 Problems

Putnam 2015/B1. Let f be a three times differentiable function (defined on $\mathbb{R}$ and real-valued) such that f has at least five distinct real zeros. Prove that $f + 6f' + 12f'' + 8f'''$ has at least two distinct real zeros.

Putnam 2015/B2. Given a list of the positive integers $1, 2, 3, 4, \ldots$, take the first three numbers $1, 2, 3$ and their sum 6 and cross all four numbers off the list. Repeat with the three smallest remaining numbers $4, 5, 7$ and their sum 16. Continue in this way, crossing off the three smallest remaining numbers and their sum, and consider the sequence of sums produced: $6, 16, 27, 36, \ldots$. Prove or disprove that there is some number in the sequence whose base 10 representation ends with 2015.

Putnam 2015/B3. Let S be the set of all 2×2 real matrices

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

whose entries a, b, c, d (in that order) form an arithmetic progression. Find all matrices M in S for which there is some integer $k > 1$ such that M^k is also in S.