

Putnam E.14

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1 Problems

Putnam 2015/A1. Let A and B be points on the same branch of the hyperbola $xy = 1$. Suppose that P is a point lying between A and B on this hyperbola, such that the area of the triangle APB is as large as possible. Show that the region bounded by the hyperbola and the chord AP has the same area as the region bounded by the hyperbola and the chord PB .

Putnam 2015/A2. Let $a_0 = 1$, $a_1 = 2$, and $a_n = 4a_{n-1} - a_{n-2}$ for $n \geq 2$. Find an odd prime factor of a_{2015} .

Putnam 2015/A3. Compute

$$\log_2 \left(\prod_{a=1}^{2015} \prod_{b=1}^{2015} (1 + e^{2\pi i ab/2015}) \right)$$

Here i is the imaginary unit (that is, $i^2 = -1$).