

Putnam E.10

Po-Shen Loh

29 October 2019

1 Problems

Putnam 2014/A1. Prove that every nonzero coefficient of the Taylor series of

$$(1 - x + x^2)e^x$$

about $x = 0$ is a rational number whose numerator (in lowest terms) is either 1 or a prime number.

Putnam 2014/A2. Let A be the $n \times n$ matrix whose entry in the i -th row and j -th column is

$$\frac{1}{\min(i, j)}$$

for $1 \leq i, j \leq n$. Compute $\det(A)$.

Putnam 2014/A3. Let $a_0 = 5/2$ and $a_k = a_{k-1}^2 - 2$ for $k \geq 1$. Compute

$$\prod_{k=0}^{\infty} \left(1 - \frac{1}{a_k}\right)$$

in closed form.