Putnam E.2

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1 Problems

2012/A1. Let $d_1, d_2, \ldots, d_{12}$ be real numbers in the open interval $(1, 12)$. Show that there exist distinct indices i, j, k such that d_i, d_j, d_k are the side lengths of an acute triangle.

2012/A2. Let $*$ be a commutative and associative binary operation on a set S. Assume that for every x and y in S, there exists z in S such that $x * z = y$. (This z may depend on x and y.) Show that if a, b, c are in S and $a * c = b * c$, then $a = b$.

2012/A3. Let $f : [-1, 1] \to \mathbb{R}$ be a continuous function such that

(i) $f(x) = \frac{2-x^2}{2} f\left(\frac{x^2}{2-x^2}\right)$ for every x in $[-1, 1]$,

(ii) $f(0) = 1$, and

(iii) $\lim_{x \to 1^-} \frac{f(x)}{\sqrt{1-x}}$ exists and is finite.

Prove that f is unique, and express $f(x)$ in closed form.