Putnam E.9

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1 Problems

Putnam 2001/A1. Consider a set S and a binary operation $*$, i.e., for each $a, b \in S$, $a * b \in S$. Assume $(a * b) * a = b$ for all $a, b \in S$. Prove that $a * (b * a) = b$ for all $a, b \in S$.

Putnam 2001/A2. You have coins $C_1, C_2, \ldots, C_n$. For each k, C_k is biased so that, when tossed, it has probability $1/(2k + 1)$ of falling heads. If the n coins are tossed, what is the probability that the number of heads is odd? Express the answer as a rational function of n.

Putnam 2001/A3. For each integer m, consider the polynomial

$$P_m(x) = x^4 - (2m + 4)x^2 + (m - 2)^2.$$

For what values of m is $P_m(x)$ the product of two non-constant polynomials with integer coefficients?