Putnam E.5

Po-Shen Loh

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1 Problems

Putnam 2003/A1. Let n be a fixed positive integer. How many ways are there to write n as a sum of positive integers, $n = a_1 + a_2 + \cdots + a_k$, with k an arbitrary positive integer and $a_1 \le a_2 \le \cdots \le a_k \le a_1 + 1$? For example, with $n = 4$ there are four ways: 4, 2+2, 1+1+2, 1+1+1+1.

Putnam 2003/A2. Let $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ be nonnegative real numbers. Show that

$$(a_1 a_2 \cdots a_n)^{1/n} + (b_1 b_2 \cdots b_n)^{1/n}$$
$$\le [(a_1 + b_1)(a_2 + b_2) \cdots (a_n + b_n)]^{1/n}.$$

Putnam 2003/A3. Find the minimum value of

$$|\sin x + \cos x + \tan x + \cot x + \sec x + \csc x|$$

for real numbers x.