Putnam Σ.5

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1 Problems

Putnam 1992/A4. Let f be an infinitely differentiable real-valued function defined on the real numbers. If

$$f\left(\frac{1}{n}\right) = \frac{n^2}{n^2+1}, \qquad n = 1, 2, 3, \ldots,$$

compute the values of the derivatives $f^{(k)}(0), k = 1, 2, 3, \ldots$.

Putnam 1992/A5. For each positive integer n, let $a_n = 0$ (or 1) if the number of 1's in the binary representation of n is even (or odd), respectively. Show that there do not exist positive integers k and m such that

$$a_{k+j} = a_{k+m+j} = a_{k+2m+j},$$

for $0 \leq j \leq m-1$.

Putnam 1992/A6. Four points are chosen at random on the surface of a sphere. What is the probability that the center of the sphere lies inside the tetrahedron whose vertices are at the four points? (It is understood that each point is independently chosen relative to a uniform distribution on the sphere.)