Putnam E.8

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1 Problems

Putnam 2009/A1. Let f be a real-valued function on the plane such that for every square $ABCD$ in the plane, $f(A) + f(B) + f(C) + f(D) = 0$. Does it follow that $f(P) = 0$ for all points P in the plane?

Putnam 2009/A2. Functions f, g, h are differentiable on some open interval around 0 and satisfy the equations and initial conditions

$$f' = 2f^2gh + \frac{1}{gh}, \quad f(0) = 1,$$
$$g' = fg^2h + \frac{4}{fh}, \quad g(0) = 1,$$
$$h' = 3fgh^2 + \frac{1}{fg}, \quad h(0) = 1.$$

Find an explicit formula for $f(x)$, valid in some open interval around 0.

Putnam 2009/A3. Let d_n be the determinant of the $n \times n$ matrix whose entries, from left to right and then from top to bottom, are $\cos 1, \cos 2, \ldots, \cos n^2$. (For example,

$$d_3 = \begin{vmatrix} \cos 1 & \cos 2 & \cos 3 \\ \cos 4 & \cos 5 & \cos 6 \\ \cos 7 & \cos 8 & \cos 9 \end{vmatrix}.$$

The argument of cos is always in radians, not degrees.) Evaluate $\lim_{n\to\infty} d_n$.