Putnam E.03

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1 Problems

Putnam 2006/B1. Show that the curve $x^3 + 3xy + y^3 = 1$ contains only one set of three distinct points A, B, and C, which are vertices of an equilateral triangle, and find its area.

Putnam 2006/B2. Prove that for every set $X = \{x_1, \ldots, x_n\}$ of real numbers, there exists a non-empty subset S of X and an integer m such that

$$\left| m + \sum_{s \in S} s \right| \leq \frac{1}{n+1}.$$

Putnam 2006/B3. Let S be a finite set of points in the plane. A linear partition of S is an unordered pair $\{A, B\}$ of subsets of S such that $A \cup B = S$, $A \cap B = \emptyset$, and A and B lie on opposite sides of some straight line disjoint from S (A or B may be empty). Let L_S be the number of linear partitions of S. For each positive integer n, find the maximum of L_S over all sets S of n points.