Putnam E.12

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1 Problems

Putnam 1982/A1. Let V be the region in the Cartesian plane consisting of all points (x, y) satisfying the simultaneous equations

$$|x| \leq y \leq |x| + 3 \quad \text{and} \quad y \leq 4\,.$$

Find the centroid $(\overline{x}, \overline{y})$ of V.

Putnam 1982/A2. For positive reals x, let

$$B_n(x) = 1^x + 2^x + 3^x + \cdots + n^x\,.$$

Prove or disprove the convergence of

$$\sum_{n=2}^{\infty} \frac{B_n(\log_n 2)}{(n \log_2 n)^2}\,.$$

Putnam 1982/A3. Evaluate

$$\int_0^{\infty} \frac{\tan^{-1}(\pi x) - \tan^{-1}(x)}{x} dx\,.$$