Putnam Σ.5

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1 Problems

Putnam 1983/A4. Prove that for $m \equiv 5 \pmod 6$,

$$\binom{m}{2} - \binom{m}{5} + \binom{m}{8} - \binom{m}{11} + \cdots - \binom{m}{m-6} + \binom{m}{m-3} \neq 0.$$

Putnam 1983/A5. Does there exist a positive real number α such that $\lfloor \alpha^n \rfloor - n$ is even for all positive integers n?

Putnam 1983/A6. Let T be the triangle with vertices $(0,0)$, $(a,0)$, and $(0,a)$. Find

$$\lim_{a \to \infty} a^4 e^{-a^3} \int_T e^{x^3+y^3} dxdy.$$