Putnam $\Sigma.2$

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1 Problems

Putnam 1984/A4. A convex pentagon inscribed in a circle radius 1 has two perpendicular diagonals which intersect inside the pentagon. What is the maximum area the pentagon can have?

Putnam 1984/A5. Let V be the pyramidal region $x, y, z \geq 0$, $x + y + z \leq 1$. Evaluate

$$\int_V xy^9 z^8 (1 - x - y - z)^4 dxdydz.$$

Putnam 1984/A6. Let $f(n)$ be the last non-zero digit in the decimal representation of $n!$. Show that for distinct integers $a_i \geq 0$, $f(5^{a_1} + 5^{a_2} + \cdots + 5^{a_r})$ depends only on the sum $a_1 + \cdots + a_r = a$. Write the value as $g(a)$. Find the smallest period for g, or show that it is not periodic.