

# Putnam $\Sigma.5$

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## 1 Problems

**Putnam 1995/B4.** Evaluate

$$\sqrt[8]{2207 - \frac{1}{2207 - \frac{1}{2207 - \dots}}}$$

Express your answer in the form  $\frac{a+b\sqrt{c}}{d}$ , where  $a, b, c, d$  are integers.

**Putnam 1995/B5.** A game starts with four heaps of beans, containing 3, 4, 5, and 6 beans. The two players move alternately. A move consists of taking **either**

- a) one bean from a heap, provided at least two beans are left behind in that heap, **or**
- b) a complete heap of two or three beans.

The player who takes the last heap wins. To win the game, do you want to move first or second? Give a winning strategy.

**Putnam 1995/B6.** For a positive real number  $\alpha$ , define

$$S(\alpha) = \{\lfloor n\alpha \rfloor : n = 1, 2, 3, \dots\}.$$

Prove that  $\{1, 2, 3, \dots\}$  cannot be expressed as the disjoint union of three sets  $S(\alpha), S(\beta)$  and  $S(\gamma)$ . As usual,  $\lfloor x \rfloor$  is the greatest integer  $\leq x$ .