

# Putnam E.14

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## 1 Problems

**Putnam 1981/B1.** Evaluate

$$\lim_{n \rightarrow \infty} \frac{1}{n^5} \sum_{r=1}^n \sum_{s=1}^n (5r^4 - 18r^2s^2 + 5s^4).$$

**Putnam 1981/B2.** What is the minimum value of  $(a-1)^2 + (\frac{b}{a}-1)^2 + (\frac{c}{b}-1)^2 + (\frac{4}{c}-1)^2$ , over all real numbers  $a, b, c$  satisfying  $1 \leq a \leq b \leq c \leq 4$ .

**Putnam 1981/B3.** Prove that infinitely many positive integers  $n$  have the property that for any prime  $p$  dividing  $n^2 + 3$ , we can find an integer  $m$  such that  $p$  divides  $m^2 + 3$  and  $m^2 < n$ .