Putnam E.13

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1 Problems

Putnam 1981/A1. Let the largest power of 5 dividing $1^1 2^2 3^3 \cdots n^n$ be $5^{f(n)}$. What is $\lim_{n\to\infty} f(n)/n^2$?

Putnam 1981/A2. We can label the squares of an 8×8 chess board from from 1 to 64 in 64! different ways. For each way, we calculate D, the largest difference between the labels of two squares which are adjacent (orthogonally or diagonally). What is the smallest possible D?

Putnam 1981/A3. Evaluate:

$$\lim_{k\to\infty} e^{-k} \int_R \frac{e^x - e^y}{x - y} dxdy\,,$$

where R is the rectangle $0 \leq x, y \leq k$.