

Putnam E.12

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14 Nov 2012

1 Problems

Putnam 1982/B1. ABC is an arbitrary triangle, and M is the midpoint of BC . How many pieces are needed to dissect AMB into triangles which can be reassembled to give AMC ?

Putnam 1982/B2. Let $a(r)$ be the number of lattice points inside the circle centered at the origin with radius r . Let $k = 1 + e^{-1} + e^{-4} + \cdots + e^{-n^2} + \cdots$. Express

$$\int_U a(\sqrt{x^2 + y^2}) e^{-(x^2 + y^2)} dx dy$$

as a polynomial in k , where U represents the entire plane.

Putnam 1982/B3. Let p_n be the probability that two numbers selected independently and randomly from $\{1, 2, 3, \dots, n\}$ have a sum which is a square. Find $\lim_{n \rightarrow \infty} p_n \sqrt{n}$.