Putnam E.8

Po-Shen Loh

17 Oct 2012

1 Problems

Putnam 1984/B1. Define $f(n) = 1! + 2! + \cdots + n!$. Find a recurrence relation $f(n+2) = a(n)f(n+1) + b(n)f(n)$, where $a(x)$ and $b(x)$ are polynomials.

Putnam 1984/B2. Find the minimum of $f(x, y) = (x - y)^2 + (\sqrt{2 - x^2} - \frac{9}{y})^2$ in the half-infinite strip $0 < x < \sqrt{2}$, $y > 0$.

Putnam 1984/B3. Let S be a set with n elements. Can we find a binary operation $\star$ on S which satisfies (1) right cancellation: $a \star c = b \star c$ implies $a = b$ (for all a, b, c), and (2) total non-associativity: $a \star (b \star c) \neq (a \star b) \star c$ for all a, b, c? Note that we are not just requiring that $\star$ is not associative, but that it is never associative.