2011-10-17 (1)

VTRMC 2006/5

Decreasing. But can rearrange 3's.

Alt. sum converges?

$- + - | + - + | - + - | + - + | - + - | + - + | \cdots$

equal. Sh. down

What if it was $- + | - + | - + |$.

and we kept getting gain? Then we increasing gaps but monotone decr anyway. Won't make difference

Rearrange the $(-1)^k$ instead

decreasing sequence

$+ \underbrace{- - | + +}_{\substack{\text{equal}\\ \le 0}} \underbrace{- | +}_{\substack{\text{equal}\\ \le 0}} \underbrace{- - | + +}_{\substack{\text{equal}\\ \le 0}} \underbrace{- | +}_{\substack{\text{equal}\\ \le 0}} \underbrace{- - | + +}_{\substack{\text{equal}\\ \le 0}} - |$ so still converges

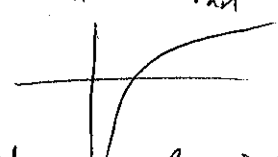
So still converges

VTRMC 2004/7

Cumulative decrease? $\left(1-\frac{a_{n+1}}{a_n}\right)^{\dagger}\left(1-\frac{a_{n+2}}{a_{n+1}}\right)$ with r_n, r_{n+1}

$\lim_{n\to\infty} a_n = 0.$ $\quad \sum \left|1-\frac{a_{n+1}}{a_n}\right|$ diverges?

% change



$L_n = \log a_n$

$\lim_{n\to\infty} L_n = -\infty.$

Recall $\log(x) \le x-1.$

$-\log x \ge 1-x$

$a_n \to a_{n+1}\left(\times \frac{a_{n+1}}{a_n}\right) = 1-r_n.$

$\cancel{\log} \frac{a_{n+1}}{a_n} - 1 \ge \log \frac{a_{n+1}}{a_n} = \log a_{n+1} - \log a_n$

$= L_{n+1} - L_n$

$a_{n+1} \to a_{n+2}\left(\times\, 1-r_{n+1}\right)$

So $a_n \to a_{n+2}: \left(\times\,(1-r_n)(1-r_{n+1})\right)$

$\ge 1 - r_n + r_{n+1}$

Show $\sum_{\oplus}\left(1-\frac{a_{n+1}}{a_n}\right) \to +\infty.$

~~even with the occasional $\ominus$ term~~ N falls to $\frac{1}{2}$

$1-\frac{a_{n+k}}{a_n} \le r_n + r_{n+1} + \cdots + r_{n+k}$

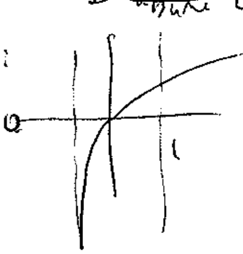
If $\exists\ \infty$ many $1-\frac{a_{n+1}}{a_n} > \frac{1}{2}$, then dvs.

So look beyond these, still sum. Now we have $1-2x \ge \log x.$

$2x-1 \le \log x$ etc.

Know $\log(x+1)$:

for $-1<x<0$



$\log(x+1)$

1

2011-10-18 ④

VTRMC 2004/7

Let $\frac{a_{n+1}}{a_n} = 1 - r_n$. We will show $\sum_{\oplus \text{ terms}} 1 - \frac{a_{n+1}}{a_n} = +\infty$

$$= \sum_{\oplus r_n} r_n = +\infty$$

Note: $\cancel{\frac{a_n}{a_n}}$ $\frac{a_{n+2}}{a_n} = \frac{a_{n+2}}{a_{n+1}} \frac{a_{n+1}}{a_n} = (1-r_{n+1})(1-r_n) \geq 1 - r_{n+1} - r_n$.

~~Wait, if any $r_n < 0$, then can replace with~~ ← Suppose all r_n's ⊕

Eventually, $a_{n+2} < \frac{1}{2} a_n$, so then find sum of finite chains of r_n's that is $> \frac{1}{2}$

Now if some r_n's negative

$$\frac{1}{2} > \underset{\substack{\text{finally}\\ <\frac{1}{2}}}{\frac{a_{n+5}}{a_n}} = \frac{a_{n+5}}{a_{n+4}} \frac{a_{n+4}}{a_{n+3}} \overset{>1}{\frac{a_{n+3}}{a_{n+2}}} \frac{a_{n+2}}{a_{n+1}} \frac{a_{n+1}}{a_n} > (1-r_{n+1})(1-r_{n+3})(1-r_{n+4})(1-r_n)$$

induction.

$$\geq 1 - r_{n+1} + r_{n+3} + r_{n+4} + r_n$$

still ok

VTRMC 2002/7

GM²: $b_n \leq \frac{a_1 + \dots + a_n}{n}$ then square

$(a_1+a_2+a_3)^2 \leq 3 \cdot (a_1^2+a_2^2+a_3^2)$

AM ≤ RMS

$$\left(\frac{x+y+z}{3}\right)^2 \leq \frac{x^2+y^2+z^2}{3}$$

$$a_1^2 + \left(\frac{a_1+a_2}{2}\right)^2 + \left(\frac{a_1+a_2+a_3}{3}\right)^2 + \left(\frac{a_1+a_2+a_3+a_4}{4}\right)^2 + \dots$$

$$\leq a_1^2 + \left(\frac{a_1^2+a_2^2}{2}\right) + \left(\frac{a_1^2+a_2^2+a_3^2}{3}\right) + \frac{a_1^2+a_2^2+a_3^2+a_4^2}{4} + \dots$$

$$= a_1^2\left[1 + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4}\right]$$ Not bounded

$$+ a_2^2\left[\tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4}\right]$$

bdd by $\frac{\pi^2}{6}$

or straight: $a_1^2\left(1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \dots\right]$ ok

$$S_n \leq \begin{array}{l} \\ + a_2^2\left[\frac{1}{4} + \frac{1}{9} + \dots\right] \\ + a_1 a_3\left[2 \cdot \left(\frac{1}{4} + \frac{1}{9} + \dots\right]\right. \end{array}$$ ← $\frac{\pi^2}{3}$

$$\leq \frac{\pi^2}{6}\left[(a_1 + \dots + a_n)^2\right] ✓$$