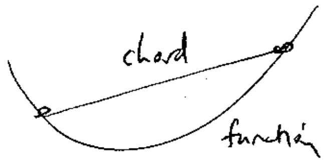
PUTNAM   Contradiction.

2011-08-30

(E).

[1] Convex:



always lies above. Weighted Mean.

Say:

Slope decreasing. ✱

then chord below function.

[2]. Assume $e = \frac{a}{b}$, integers.

$e = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots = \frac{a}{b}$.

Then $\left[\frac{a}{b} - \left(\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\cdots+\frac{1}{b!}\right)\right] b!$ is an integer

So $x = \left[e - \frac{1}{0!} - \frac{1}{1!} - \cdots - \frac{1}{b!}\right] b!$ is integer

$= b!\cdot\left(\frac{1}{(b+1)!}+\frac{1}{(b+2)!}+\cdots\right)$ is integer.

Clearly > 0.

But it's also: $\frac{1}{b+1}+\frac{1}{(b+2)(b+1)}+\frac{1}{(b+3)(b+2)(b+1)}+\cdots$

$\le \frac{1}{(b+1)}+\frac{1}{(b+1)^2}+\frac{1}{(b+1)^3}+\cdots$

$= \frac{\frac{1}{(b+1)}}{1-\frac{1}{b+1}} = \frac{1}{b} < 1.$ ✱

2011-08-30

(F)

$\boxed{\text{HUN 99}}$. Let $p_1, p_2, \cdots, p_k$ be the primes.

Get: $p_1^{a} p_2^{a} \cdots p_k^{a}$ as the first number $\rightarrow$ ~~[illegible]~~ v_1 where 1st coord is power of p_1 etc.

$\vdots$

$p_1^{a} \cdots p_k^{a}$ as the $(k+1)$st number $\longrightarrow$ v_{k+1}

Condition: For v_1, there is ~~[illegible]~~ coordinate where it exceeds the sum of other $v_{i \neq 1}$.

Pigeonhole: Some time coordinate used twice, say WLOG, by v_1 and v_2. 1st coord, and by $\divideontimes$

ALT: Linear algebra. There is nontrivial $\sum c_i \vec{v}_i = \vec{0}$.

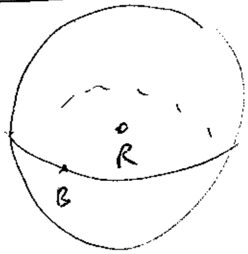
Take maximum absolute value c_i. WLOG it is c_1, and divide it out.

Get: $\vec{v}_1 = c_2' \vec{v}_2 + c_3' \vec{v}_3 + \cdots + c_n' \vec{v}_n$. all $|c_i'| \leq 1$.

But as vectors, componentwise $\leq \vec{v}_2 + \vec{v}_3 + \cdots + \vec{v}_n$. $\divideontimes$

$\boxed{\text{GER 85.}}$ Suppose R misses ~~all~~ some distance, ~~say~~ d_R

If no R at all, still have this sphere



Get B+G sphere radius d_R.

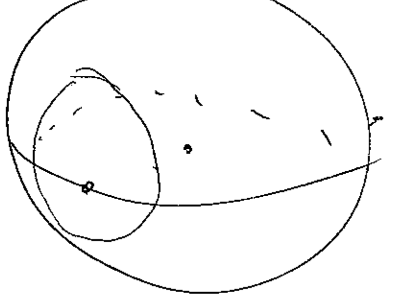
Say B misses some $d_B < 2d_R$

Then get entire circle of G. $\Rightarrow$ all dists inside it.

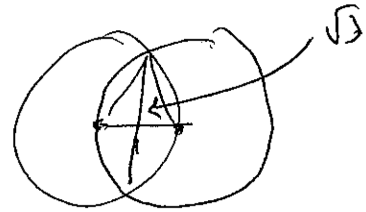
So take ~~biggest~~ big missing distance d_R.

Say R, G, B all missing distances $d_R \geq d_B \geq d_G$

Get:



diameter. ~~Radius~~ at least:



So can get G, continuous circle with diameter $\geq \sqrt{3}\, d_G$. $\divideontimes$