

# Even more advanced Putnam training

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## 1 Problems

**Putnam 1998/B1.** Find the minimum value of

$$\frac{(x + 1/x)^6 - (x^6 + 1/x^6) - 2}{(x + 1/x)^3 + (x^3 + 1/x^3)}$$

for  $x > 0$ .

**Putnam 1995/A2.** For what pairs  $(a, b)$  of positive real numbers does the improper integral

$$\int_b^\infty \left( \sqrt{\sqrt{x+a} - \sqrt{x}} - \sqrt{\sqrt{x} - \sqrt{x-b}} \right) dx$$

converge?

**Putnam 1996/A3.** Suppose that each of 20 students has made a choice of anywhere from 0 to 6 courses from a total of 6 courses offered. Prove or disprove: there are 5 students and 2 courses such that all 5 have chosen both courses or all 5 have chosen neither course.