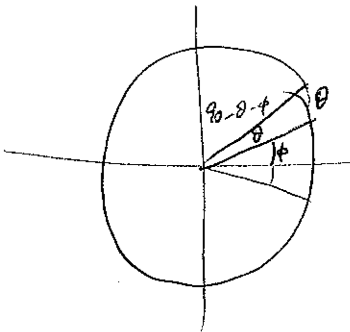
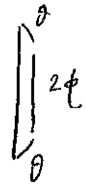
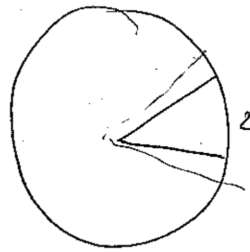
2010-10-20

①

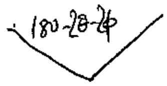
1998/A2



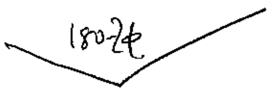




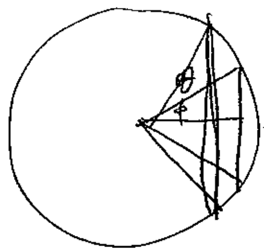
$2(\theta+\phi)$

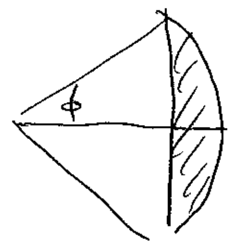


$90-\theta-\phi$



$90-\phi$





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So: $(\theta+\phi) - \sin(\theta+\phi)\cos(\theta+\phi)$

$+ [\phi - \sin\phi\cos\phi$

$$= \pi\frac{2\phi}{2\pi} - \frac{1}{2}\sin 2\phi.$$

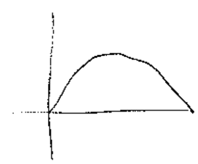
$$\left[\begin{array}{l}(\theta+\phi) - \frac{1}{2}\sin(2(\theta+\phi)) \\ -\phi + \frac{1}{2}\sin 2\phi\end{array}\right]$$

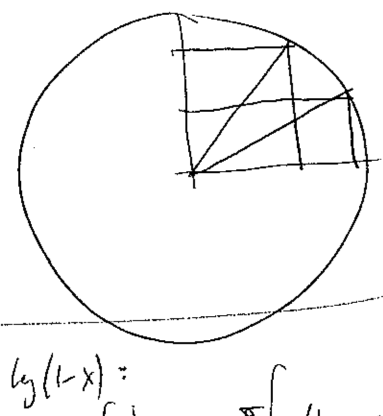
$$= \phi - \frac{1}{2}\sin 2\phi$$

$$= \phi - \sin\phi\cos\phi.$$

$$+\left[\begin{array}{l}(\overset{90-\phi}{\cancel{180-2\phi}}) - \frac{1}{2}\sin(180-2\phi) \\ -(90-\phi-\theta) + \frac{1}{2}\sin(180-2\phi-2\theta)\end{array}\right],$$

$= 2\theta.$





$1-\frac{1}{2n} > e(1-\frac{1}{n})^n > 1-\frac{1}{n}$

$\lg(1-\frac{1}{2n})$ $\lg(1-\frac{1}{n})$

$\underbrace{1+n\lg(1-\frac{1}{n})}$

$1+n(-\frac{1}{n}-\frac{1}{2n^2}-\frac{1}{3n^3}\cdots)$

$= \phi - \frac{1}{2n} - \frac{1}{3n^2} - \frac{1}{4n^3}\cdots$

Taylor $\lg(1-x)$:

$-\int\frac{dx}{1-x} = \int -(1+x+x^2+x^3+\cdots)$

$= -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4}\cdots$

$-\frac{1}{2n} - \frac{1}{2\cdot 2^2 n} - \frac{1}{3\cdot 2^3 n}$

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(2)

$s+1 \quad s+(n-s)$

$\{ \underbrace{1\,2\,3\,4\,5 \ldots s-1}_{No} \; s \; \frac{\text{any}}{} \}$. size $= s$.

$\binom{n-s}{s-1}$.

$$\sum_{s=1}^{n/2} \binom{n-s}{s-1}$$

$$\binom{n-1}{0} + \binom{n-2}{1} + \binom{n-3}{2} + \binom{n-4}{3} + \binom{n-5}{4} + \ldots$$

$n=3$: $\binom{2}{0} + \binom{1}{1} = 1 + 1 = 2$. F_2

$n=4$: $\binom{3}{0} + \binom{2}{1} = 1 + 2 = 3$. F_3

$n=5$: $\binom{4}{0} + \binom{3}{1} + \binom{2}{2} = 1 + 3 + 1 = 5$.

$n=6$: $\binom{5}{0} + \binom{4}{1} + \binom{3}{2} = 1 + 4 + 3 = 8$.

$\longrightarrow$ Fibonacci.

s dominoes.
$n-2s$ squares
$n-s$ total objects.

Tilings with □ or ◫.

$$= \sum_{t=0}^{\infty} \binom{n-t-1}{t} = \text{tile } n-1 \text{ with } □, ◫.$$

$t=s-1$
$s=t+1$

$F_0 = 1$
F_1 □ $\to 1$
F_2 ◫, □□ $\to 2$ ✓.

F_{n-1}

2002/B3

Fix $\underline{k}$.

$f(x) = k\left[e^{-kx} - (1-x)^k\right]$

$f'(x) = k\left[-ke^{-kx} + k(1-x)^{k-1}\right]$

So $e^{-kx} = (1-x)^{k-1}$.

@ there, $f(x) = k\left[e^{-kx} - e^{-kx}(1-x)\right]$

$= xk\, e^{-kx}$. $\quad te^{-t}$ max at $\frac{1}{e}$. At $t=1$

~~works for k=~~

$(1-x)^k \geq e^{-kx} - \frac{1}{ek}$

$(1-\frac{1}{n})^n \geq e^{-1} - \frac{1}{en}$.

$\frac{1}{en} \geq \frac{1}{e} - (1-\frac{1}{n})^n$

and this is not at $x = \frac{1}{n}$:

$e^{-1} \overset{?}{=} (1-\frac{1}{n})^{n-1}$ ⊠

$\uparrow$ irrational