Dan SHAFER 48 points, rank 145.5 (junior)
Alan PIERCE 39 points, rank 225 (sophomore)
Pawat TECHAPONGTADA 31 points, rank 300 (senior)
Geoff CAMERON 30 points, rank 335.5 (junior)
Igor BALLA 28 points, rank 400.5 (freshman)
Mark SPINDLER 26 points, rank 425.5 (senior)
Emily ALLEN 22 points, rank 473 (senior)

2010-09-24

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Joe KLOBUSICKY 21 points, rank 518.5
Theodore GAST 20 points, rank 619
Brian McFARLAND, Stephen RUSSELL 12 points, rank 876.5
Brian LEARY, Vasu BALAKRISHNAN 11 points, rank 969.5
DJ STERLING, Drew BESSE, Hui Han CHIN 10 points, rank 1196.5
Tim SELF 6 points, rank 1519
Franklin TA 5 points, rank 1521
Timothy NAUMAVITZ 4 points, rank 1533.5
Ben WOLF, Kelly RIVERS 3 points, rank 1561.5
Chun How TAN 0 point, rank 2771.5

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2001/A1

$$(a*b)*a=b.$$

$$(a*a)*a=a.$$

$$a*(b*a)$$

$$(b*a)*b=a.$$

$$(\underbrace{(a*a)*a}_{\text{"}a\text{"}})*(b*a)=$$

$$= [\underbrace{(b*a)*b}_{\text{"}a\text{"}}]*\underbrace{(b*a)}_{\text{"}a\text{"}}=b.$$

$$a*(b*a)$$

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2001/B2

$$\frac{1}{x}+\frac{1}{2y}=(x^2+3y^2)(3x^2+y^2)=3x^4+10x^2y^2+3y^4.$$

$$+\quad \frac{1}{x}-\frac{1}{2y}=2y^4-2x^4$$

$$\oplus\quad \frac{2}{x}=x^4+10x^2y^2+5y^4 \longrightarrow 2=x^5+10x^3y^2+5xy^4$$

$$\ominus\quad \frac{1}{y}=5x^4+10x^2y^2+y^4 \longrightarrow 1=5x^4y+10x^2y^3+y^5.$$

$$\ominus\ 1=x^5-5x^4y+10x^3y^2-10x^2y^3+5xy^4-y^5$$

$$=(x-y)^5 \longrightarrow \cancel{\text{any } x,y \text{ s.t. } x-y=1}.$$

$$\cancel{\text{Except } 0\text{'s}}$$

$$\oplus\ 3=(x+y)^5$$

$$x-y=1.\qquad x+y=\sqrt[5]{3}$$

$$x=\frac{1+\sqrt[5]{3}}{2}\qquad y=\frac{\sqrt[5]{3}-1}{2}.$$

2001 A3

$$X^4 - (2m+4)x^2 + (m-2)^2.$$

$(x+a)(x^3+bx^2+cx+d)$?

Then $-a$ is a root, integer. What integer roots can we have? $r \mid (m-2)^2$.

Actually, $r^2 \mid (m-2)^2$.

But this is poly in $x^2$.

So can factor: (FTA) $\quad (x^2-\alpha)(x^2-\beta) = x^4 - (2m+4)x^2 + (m-2)^2$.

$$\alpha, \beta = \frac{(2m+4) \pm \sqrt{(2m+4)^2 - 4(m-2)^2}}{2}$$

$$= \frac{(2m+4) \pm \sqrt{4m^2+16m+16-4m^2+16m-16}}{2}$$

$$= \frac{2m+4 \pm \sqrt{32m}}{2}.$$

$$\alpha, \beta = m+2 \pm \sqrt{8m}. \qquad \cancel{(\sqrt{m}+\sqrt{2})}^2 = m+2+2\sqrt{2m}.$$

Certainly if $\sqrt{8m}$ is integer, then ok $\longrightarrow$ $m = 2 \times$ perfect square

$$= (\sqrt{m}+\sqrt{2})^2, \ (\sqrt{m}-\sqrt{2})^2.$$

So it is $(x+\sqrt{m}+\sqrt{2})(x-\sqrt{m}-\sqrt{2})(x+\sqrt{m}-\sqrt{2})(x-\sqrt{m}+\sqrt{2})$

Can ever $\pm\sqrt{m} \pm \sqrt{2}$ be integer?

Say $\sqrt{m}+\sqrt{2} = z$.

Then $m + 2\sqrt{2m} + 2 = z^2$.

$\Rightarrow m$ is $2\times$ perfect square, already done since (2)(2)

Say $\sqrt{m}-\sqrt{2}$ ... same argument

$(x+\sqrt{m}+\sqrt{2})(x-\sqrt{m}+\sqrt{2})$
$= x^2 + 2\sqrt{2}x + 2 - m$
Never Z.

So ALWAYS (2)(2)

Can we combine $(x+\sqrt{m}+\sqrt{2})(x+\sqrt{m}-\sqrt{2})$?

$x^2 + 2\sqrt{m}\,x + m - 2.$

$\Leftarrow$ Need $m$ perfect square.

$(x-\sqrt{m}-\sqrt{2})(x-\sqrt{m}+\sqrt{2})$
$= x^2 - 2\sqrt{m}x + m - 2.$