2010-10-26
①

VTRMC 2008/2

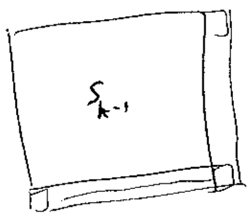
1, 3 sum to 16

$f(n) = f(n-1) + f(n-3)$

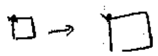
n	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$f(n)$	1	1	2	3	4	6	9	13	19	28	41	60	88	129

15: 189
16: 189 + 88 = 277

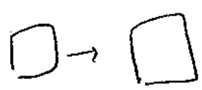
VTRMC 2001/3



$S_n = S_{n-1} + n^2$.



same corner, so ending in $(n-1)\times(n-1)$ square



$S_8 = 1^2 + 2^2 + \dots + 8^2$

$\sum^n i^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum_1^n i^3 = \left[\frac{n(n+1)}{2}\right]^2$

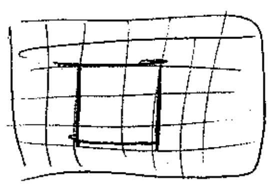
~~$(k+1)^2 - k^2 = 2k$~~

$$\left(\sum^{k+1} i\right)^2 - \left(\sum^{k} i\right)^2 = \left(\sum^{k+1} i + \sum^{k} i\right)(k+1)$$

$$\sum^{k+1} i + \sum^{k} i = \frac{(k+1)(k+2)}{2} + \frac{k(k+1)}{2}$$

$$= (k+1)(k+1) \quad (k+1)$$

rect:

n  m

$\binom{n+1}{2}\binom{m+1}{2}$

V 2004/3

no AAA

ending in B/C → 3 choices.

begin: B/C, A B/C, AA B/C

$S_n = 2S_{n-1} + 2S_{n-2} + 2S_{n-3}$

$S_1 = 3$

$S_2 = 9$

$S_3 = 27 - 1 = 26$

$S_4 = 2(3+9+26) = 76$

$S_5 = 222 \qquad S_6 = 648$

12 + 26 = 38

$$\boxed{\frac{648}{3^6} = \frac{8}{9}}$$

2010-10-26

(2)

V 2002/5 Scaife 125 9–11:30 AM

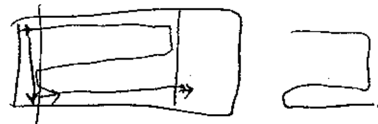
where is 1st 0?

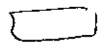
1 2 3 4 . . . — — n.

$f(1) = 1.$

0 [n-2] [n-3]

[2] 1 0. none → +1



0 

$X_{n-1} + \cdots + X_1 + 1$

0.

$f(n) = f(n-2) + \cdots + f(1) + 1$

$f(3) = 2$

$f(n-1) = f(n-3) + \cdots + f(1) + 1$

$f(4) = 3$

So $f(n) = f(n-1) + f(n-2)$ fibonacci.

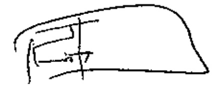
$1.7^{n-1} + 1.7^{n-2} = 1.7^n \left[\frac{1}{1.7} + \frac{1}{1.7^2}\right] = \frac{1.7+1}{1.7^2} = \frac{2.7}{2.89} < 1$

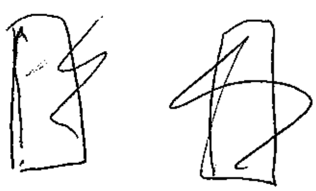
V 2008/3

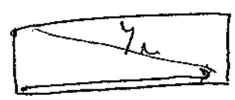
$t(1) = 2.$ 0 or 8

$t(2) =$ 08 or ▭ or ▱

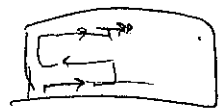
8



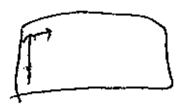




y_n x_n.

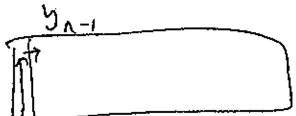


or



$y_{n-2} + \cdots$ y_{n-1}

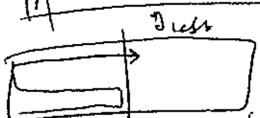
y_{n-1}



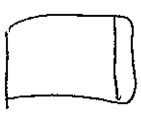
$X_n =$

$X_n = y_{n-1} + \cdots + y_1$

$X_1 = y_1 = 1,$



$X_n = 2^{n-2}$



x_1	0	1	
y	1	0	
n.	1	2	3

$y_1 = 1$

$y_n = X_{n-1} + \cdots + X_1$

$X_1 = 0,$

$X_2 = 1.$

$y_2 = 1.$

$X_3 =$

$$y_n = x_{n-1} + x_{n-2} + \dots + x_1 + 1$$

$$x_n = y_{n-1} + \dots + y_1$$

n	1	2	3	4	5	6	7
x	0	1	2	4	8	16	
y	1	1	2	4	8	16	

$$2^{n-2}$$

2010-10-28
③

2005/B4

$f(m,n)$: # $|x_1|+|x_2|+\cdots+|x_n| \leqq m$.

$x_1 = -m \longrightarrow f(m-m, n-1)$
$\quad -m+1 \longrightarrow f(m-(m-1), n-1)$
$\quad \vdots \qquad \vdots$
$\quad 0$
$\quad \vdots$
$\quad +m \quad f(m-m, n-1)$

$$= \sum_{k=1}^{m} 2 f(m-k, n-1) + f(m, n-1)$$

$\cancel{\leq m}$
$< m$: $f(m-1, n)$

$= m$, $x_n \geqq 0$. $f(m, n-1)$

$= m$, $x_n < 0$ $f(m-1, n-1)$

$$F(M,n) = \sum_{m=0}^{M} f(m, n-1)$$

$f(0, n-1) + f(1, n-1) + f(2, n-1) + \cdots + f(M, n-1)$

$+ 2f(m-1, n-1)$ ⋯

$\cdot 2f$