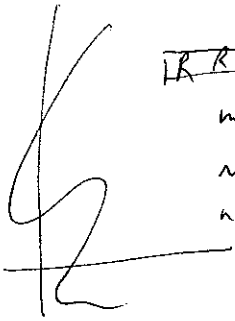
Pos # > 0 : 1 > 0

2004/ B2.

2010-10-12

①.

$$\binom{m+n}{m} < \frac{(m+n)^{m+n}}{m^m n^n}$$



m

n

R R R B B B R

m+n bins.

m R's

n B's.

# of occupancy patterns.

B R R | B | B B B

R R R B B

(R_1, B_1)

m+n bins.

m R's. Each ball can go anywhere

n B's.

$$\frac{(m+n)!}{(m+n)^{m+n}}$$

Each ball exactly once

$\frac{n!}{m!}$

all red balls first,

Don't care which

(R_1, B_2, B_3)

!

RRRRBB > RRRR BB.

w/o replacement with replacement

$$(m+n)! < \left(\left(\frac{m+n}{m}\right)^m m!\right)\left(\left(\frac{m+n}{n}\right)^n n!\right) \quad n!$$

$\frac{m!\,n!}{(m+n)!}$. RRRRRRRBBBB

$$(m+n) \times \left(\frac{m+n}{m}(m-1) = m+n - \frac{m+n}{m}\right)$$

$\cancel{\left(\frac{m+n}{n}\right)^{m+n}}$ $\frac{(m+n)^{m+n}}{m^m n^n}\, m!$

$$\left(\frac{m+n}{m}\right)^m \left(\frac{m+n}{n}\right)^n m!$$

Binomial exp of $(m+n)^{m+n}$ has $\binom{m+n}{m} m^m n^n$. (m terms)

2010-12-12

(2)

CA 104 Cauchy: Equality case:

$$\langle \|y\|x - \|x\|y, \|y\|x - \|x\|y \rangle \geq 0.$$

equality iff $\|y\|x = \|x\|y$.
so proportional.

$$n = \sum a_i \cdot 1 = (a_1, a_2, \ldots, a_n) \cdot (1, 1, 1, \ldots, 1) \leq \sqrt{\sum a_i^2} \cdot \sqrt{n}.$$

$$\sqrt{n} \leq \sqrt{\sum a_i^2}$$

intuition for smallest: all equal.

$$n \leq \sum a_i^2$$

Actually, showed: $\sum a_i \geq n \Rightarrow \sum a_i^2 \geq n.$

$$\Rightarrow \sum a_i^4 \geq n.$$

CA 105

$$\leq \sqrt{\sum a_i^2 \sum a_i'^2} = \sum a_i^2.$$

GA 109 $P(x) = c_n x^n + c_{n-1} x^{n-1} + \cdots c_0.$

$$\sqrt{P(a) P(b)} = \sqrt{(c_n a^n + \cdots + c_0)(c_n b^n + \cdots + c_0)}$$

vectors: $\sqrt{c_n}(\sqrt{a})^n + \sqrt{c_{n-1}}(\sqrt{a})^{n-1} + \cdots + \sqrt{c_0}$

$\sqrt{c_n}(\sqrt{b})^n + \cdots \sqrt{c_0}$

$\geq$ dot product: $c_n(\sqrt{ab})^n + \cdots + c_0$

2003/A2

AM-GM: $\frac{a+b}{2} \geq \sqrt{ab}.$

$\Updownarrow$

$\frac{a^2+2ab+b^2}{4} \geq ab.$

$a^2+2ab+b^2 \geq 4ab.$

$(a-b)^2 \geq 0.$

EQ: $a=b$.

$$GM(a_i) + GM(b_i) \leq GM(a_i + b_i).$$

AM(GM GM) $\leq$ GM(AM)

GM AM

$$\sqrt[n]{a_1 a_2 \cdots a_n} + \sqrt[n]{b_1 b_2 \cdots b_n} \stackrel{?}{\leq} \sqrt[n]{(a_1+b_1)\cdots(a_n+b_n)}$$

$$\sqrt[n]{\frac{a_1}{a_1+b_1} \frac{a_2}{a_2+b_2} \cdots \frac{a_n}{a_n+b_n}} + \sqrt[n]{\frac{b_1}{a_1+b_1} \cdots \frac{b_n}{a_n+b_n}} \leq 1.$$

$$GM \leq AM = \frac{1}{n}\left(\frac{a_1}{a_1+b_1} + \cdots + \frac{a_n}{a_n+b_n}\right)$$

$$\leq \frac{1}{n}\left(\frac{b_1}{a_1+b_1} + \cdots + \frac{b_n}{a_n+b_n}\right) \qquad = \frac{1}{n}(\overbrace{1 + \cdots + 1}^{n}) = 1$$

2010-10-12

AM-GM

$$\frac{a+b+c}{3} \geq \sqrt[3]{abc}.$$

Back induction: true for n, show for $n-1$.

$x_1, \dots, x_{n-1}$ given. Let μ = their AM.

Apply AM-GM on $x_1, \dots, x_{n-1}, \mu$:

$$\sqrt[n]{x_1 x_2 \cdots x_{n-1} \mu} \leq \frac{x_1 + \dots + x_{n-1} + \mu}{n} = \mu$$

$$\left[(x_1 \cdots x_{n-1})^{\frac{1}{n}} \leq \mu^{1-\frac{1}{n}} = \frac{n-1}{n} \right]^{\frac{n}{n-1}}$$

$$GM(x_1, \dots, x_{n-1}) \leq \mu. \qquad \square$$

×2: $n \to 2n$.

$x_1, \dots, x_{2n}$ given.

Apply AM-GM to $\frac{x_1+x_2}{2}, \frac{x_3+x_4}{2}, \dots, \frac{x_{2n-1}+x_{2n}}{2}$:

$$\sqrt[n]{\frac{x_1+x_2}{2} \times \cdots \times \frac{x_{2n-1}+x_{2n}}{2}} \leq \frac{1}{n}\left(\frac{x_1+x_2}{2} + \dots + \frac{x_{2n-1}+x_{2n}}{2}\right) = AM$$

$$\leq$$

$$\sqrt[n]{\sqrt{x_1x_2}\sqrt{x_3x_4} \cdots}$$

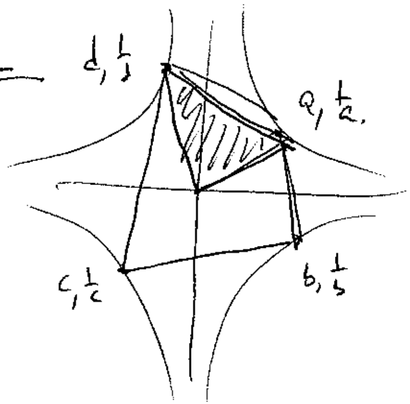
GM.

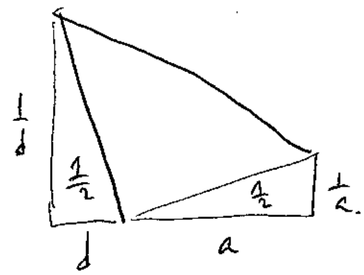
2010-10-12

4

[illegible]

<u>2007(A2</u>





$$A = (a+d)\times \tfrac{1}{2}\left(\tfrac{1}{a}+\tfrac{1}{d}\right) - 1.$$

$$= \tfrac{1}{2}\left[1 + \tfrac{a}{d} + \tfrac{d}{a} + 1\right] - 1.$$

$$= \tfrac{1}{2}\left(\tfrac{a}{d} + \tfrac{d}{a}\right].$$

<u>total Area</u>: $\frac{1}{2}\cdot\Big[\underbrace{\frac{a}{d}+\frac{d}{a}+\frac{d}{c}+\frac{c}{d}+\frac{b}{c}+\frac{c}{b}+\frac{a}{b}+\frac{b}{a}}_{\text{AM - GM.}}\Big].$

$$4\cdot\times AM\left(\tfrac{a}{d}, \ldots \tfrac{b}{a}\right).$$

$$\geq 4\times GM\left(\tfrac{a}{d} \ldots \tfrac{b}{a}\right) = 4$$