1: Given k and a k-coloring of a k-chromatic graph, prove that for any color c there is a vertex of color c which is adjacent to vertices of every other color.

2, Diestel 5.18: Given a graph G and $k \in \mathbb{N}$ let $P_G(k)$ denote the number of vertex colourings $V(G) \to \{1, \dots, k\}$. Show that P_G is a polynomial in k of degree $n := |G|$, in which the coefficient of k^n is 1 and the coefficient of k^{n-1} is $-||G||$. (P_G is called the *chromatic polynomial* of G.)
(Hint. Apply induction on $||G||$.)

3, Diestel 5.19: Determine the class of all graphs G for which $P_G(k) = k(k-1)^{n-1}$. (As in the previous exercise, let $n := |G|$, and let P_G denote the chromatic polynomial of G.)
Hint: A graph with n vertices is a tree if and only if it is connected and has $n-1$ edges.