

1, Diestel 3.5: Deduce the $k = 2$ case of Menger's theorem (3.3.1) from Proposition 3.1.1.

2, Diestel 3.17 (i): Find the error in the following 'simple proof' of Menger's theorem (3.3.1). Let X be an A - B separator of minimum size. Denote by G_A the subgraph of G induced by X and all the components of $G - X$ that meet A , and define G_B correspondingly. By the minimality of X , there can be no A - X separator in G_A with fewer than $|X|$ vertices, so G_A contains k disjoint A - X paths by induction. Similarly, G_B contains k disjoint X - B paths. Together, all these paths form the desired A - B paths in G .

3, Diestel 3.18: Prove Menger's theorem by induction on $\|G\|$, as follows. Given an edge $e = xy$, consider a smallest A - B separator S in $G - e$. Show that the induction hypothesis implies a solution for G unless $S \cup \{x\}$ and $S \cup \{y\}$ are smallest A - B separators in G . Then show that if choosing neither of these separators as X in the previous exercise gives a valid proof, there is only one easy case left to do.

4, Diestel 3.21: Let $k \geq 2$. Show that every k -connected graph of order at least $2k$ contains a cycle of length at least $2k$.

5, Diestel 3.22: Let $k \geq 2$. Show that in a k -connected graph any k vertices lie on a common cycle.
